B. Sc. I Semester Physics-I

Ch.1 Mechanics

1. Compound Pendulum: A compound pendulum is just a rigid body capable of oscillating freely about a horizontal axis passing through it. Its vibrations are simple harmonic and its time period is given by

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

Where I is the moment of inertia of a body about the axis of suspension, m is its mass, g is the acceleration due to gravity, l is length.

Expression of time period of the compound pendulum:

Let S- be the point of suspension of the rigid body through which the horizontal axis passes.

Let G be center of gravity of body which is vertically below S,

l- be length of pendulum

Let the body be displaced through an angle θ into the dotted position shown, so that C.G. is now G′.

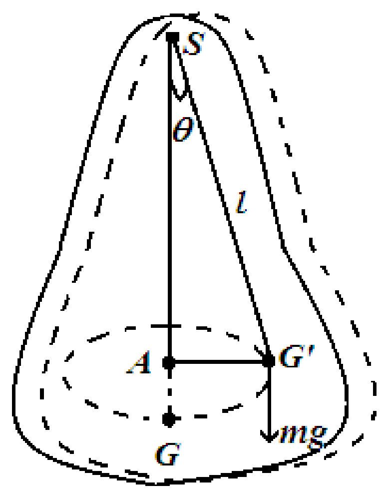


Fig. 1

During displaced position, the couple acting on the body due to its weight mg and it will be $mglsin\theta$, tending to bring it back into its original position.

∴ Restoring couple acting on the pendulum = - $mglsin\theta$

AS θ is very small then sinθ ≈ θ

∴ Restoring couple acting on the pendulum = - $mgl\theta$ ------- (1)

If angular acceleration produced in the body due to this couple be $\frac{d^2\theta}{dt^2}$, then the couple will be equal to $I\frac{d^2\theta}{dt^2}$.---------------- (2)

Where I is the moment of inertia of the body.

Equating eq^n (1) and (2)

$$I\frac{d^2\theta}{dt^2} = -mgl\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{mgl}{I}\theta \text{ ---------------------------- (3)}$$

As $\frac{mgl}{I}$ is constant then $Acc^n \propto \theta$------------------------------ (4)

From eq^n (4), it can be concluded that, the angular acceleration of the body is proportional to its angular displacement. The body executes a simple harmonic motion.

We know that, the angular acceleration of a body performing SHM is

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta \text{ --------------- (5)}$$

Comparing eq^n 4 and eq^{n}5,

$$\omega^2 = \frac{mgl}{I} \text{ and } \omega = \sqrt{\frac{mgl}{I}}$$

The time period of a body performing SHM is given by

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{I}{mgl}} \text{ -------------------------- (6)}$$

This is expression of time period of compound pendulum.

If Io be moment of inertia of the body about an axis passing through G and parallel to the axis of suspension through S,

We have, Principle of parallel axis, $I = Io + ml^2$

If K is the radius of gyration of the body about the axis through G, then $Io = mK^2$

$$\therefore \quad I = mK^2 + ml^2$$

By substituting value of I in eqn (6)

$$\therefore T = 2\pi\sqrt{\frac{mK^2+ml^2}{mgl}}$$

$$T = 2\pi\sqrt{\frac{m(K^2 + l^2)}{mgl}}$$

$$T = 2\pi\sqrt{\frac{K^2 + l^2}{gl}}$$

This is expression of time period of compound pendulum in terms of radius of gyration.

2. Interchangeability of Centers of Suspension and center of oscillation:

Let S be center of suspension and O be center of oscillation of compound pendulum.

A point O is at distance of K^2/l from center of gravity G. Thus GO = K^2/l putting it equal to l′,

We have SO = $l + l' = l + K^2/l$

Time period of compound pendulum is

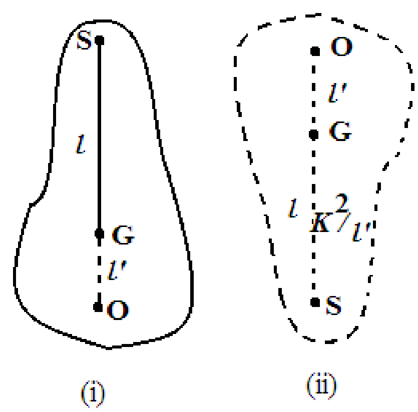


Fig 1

$$T = 2\pi\sqrt{\frac{K^2 + l^2}{gl}}$$

$$T = 2\pi\sqrt{\frac{K^2/l + l}{g}}$$

$$T = 2\pi\sqrt{\frac{l+l'}{g}} = 2\pi\sqrt{\frac{L}{g}} \text{ ---------------------------- (1)}$$

Where $L = l + l'$ is length of pendulum. Thus, the point of oscillation O lies at distance L from the point of suspension (S) gives length of simple pendulum.

If the pendulum is inverted and suspended about the axis of oscillation through O as shown in Fig. (ii), at that time, center of oscillation is S and center of suspension is O.

The time period of vibration about O is

$$T = 2\pi\sqrt{\frac{K^2 + l'^2}{gl'}}$$

Since, $K^2/l = l'$, we have $K^2 = l*l'$

Therefore, the expression of time period becomes,

$$T = 2\pi\sqrt{\frac{l.l' + l'^2}{gl'}}$$

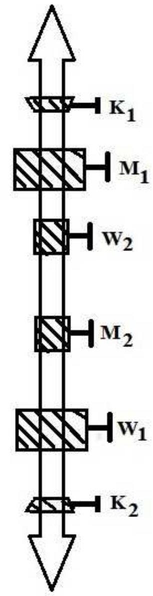
$$T = 2\pi\sqrt{\frac{l+l'}{g}} = 2\pi\sqrt{\frac{L}{g}} \text{ --------------------(2)}$$

From eq^n (1) and eq^n (2), time period is same about S and O. This shows that by interchanging center of suspension and center of oscillation, there is no change in time period of compound pendulum. This is called as interchangeability of center of suspension and center of oscillation.

3. Keter's Reversible Pendulum:

Keter's pendulum consists of a long uniform metal rod of circular cross-section.

Let M_1 is heavy metal cylinder and M_2 is small metal disc. Let W_1 is big wooden cylinder and W_2 is small wooden disc. All these cylinders can be displaced conveniently along the rod and can be set in proper position. K_1 and K_2 are movable knife edges.



It is called as reversible pendulum because it can be oscillated first by suspending on knife edge K_1 and then it can be reversed and suspended about K_2.

Working of keter's pendulum is based on the theory of compound pendulum. Thus it is basically compound pendulum with an arrangement to set proper position of its center of gravity. When position of center of gravity is properly set the pendulum swings exactly the same period about knife edges K_1 and K_2.

Suppose pendulum is suspended about K_1 and time period T_1 is measured by counting time for 50 oscillations. Then pendulum is reversed and suspended about K_2 and time period T_2 is again measured. Initially there will be difference between T_1 and T_2. Hence metal cylinder M_1 is shifted by few centimeter up or down as required. This will slightly shift position of C.G. of pendulum. T_1 and T_2 are again measured. If difference between them decreases M_1 is again shifted by few centimeters in same direction as before. If difference in T_1 and T_2 is increased, the direction of shifting of M_1 is reversed. This procedure is continued and help of W_1 is also taken in the process. It is found after few trials $T_1 = T_2$. When set, finally periodic times T_1 and T_2 will be exactly equal.

Thus when T_1 and T_2 exactly equal K_1 is center of suspension and K_2 is center of oscillation. Distance between K_1 and K_2 is measured. Suppose this distance will be *L*.

If T_1 and T_2 are differencing, average is taken as, $T = \frac{T_1+T_2}{2}$

Then $T = 2\pi\sqrt{L/g}$

To calculate *g*, squaring on both sides, $T^2 = 4\pi^2 \frac{L}{g}$

$$g = 4\pi^2 \frac{L}{T^2}$$

By using this equation acceleration due to gravity can be calculated.

4. Bessel's Contribution-computed time:

To make exactly equal time periods T_1 and T_2 about two knife edges require longer time. Bessel showed that it is not necessary to make periods T_1 and T_2 exactly equal. It is enough even if they are approximately equal.

Let T_1 be time period about K_1

Let *l* be distance of center of gravity of pendulum from K_1

$$T_1 = 2\pi\sqrt{\frac{K^2+l^2}{l.g}} \text{ ----------------(1)}$$

Squaring on both sides and rearranging,

$$T_1^2 = 4\pi^2.\frac{(K^2+l^2)}{l.g}$$

$$T_1^2 lg = 4\pi^2(K^2+l^2)\text{---------------- (2)}$$

Let T_2 be time period about K_2 and l_1 be distance of center of gravity of pendulum from K_2

$$T_2 = 2\pi\sqrt{\frac{K^2+l_1^2}{l_1.g}} \text{ ----------------(3)}$$

Squaring on both sides and rearranging,

$$T_2^2 = 4\pi^2.\frac{(K^2+l_1^2)}{l_1.g}$$

$$T_2^2 l_1.g = 4\pi^2.(K^2+l_1^2) \text{----------------- (4)}$$

Subtracting eq^n(4) from eq^n (3),

$$T_1^2 lg - T_2^2 l_1 g = 4\pi^2(K^2+l^2) - 4\pi^2(K^2+l_1^2)$$

$$g(T_1^2 l - T_2^2 l_1) = 4\pi^2(K^2+l^2-K^2-l_1^2)$$

$$= 4\pi^2\,(l^2-l_1^2)$$

$$= 4\pi^2(l+l_1)(l-l_1\,)$$

$$\frac{4\pi^2(l+l')}{g} = \frac{T_1^2 l - T_2^2 l_1}{l-l_1} = \frac{2T_1^2 l - 2T_2^2 l_1}{2(l-l_1)}$$

$$= \frac{T_1^2 l + T_1^2 l - T_2^2 l_1 - T_2^2 l_1}{2(l-l_1)}$$

Adding and subtracting $T_1^2 l_1$ and $T_2^2 l$ in the numerator.

$$\frac{4\pi^2(l+l_1)}{g} = \frac{T_1^2 l + T_1^2 l - T_2^2 l_1 - T_2^2 l_1 + T_1^2 l_1 - T_1^2 l_1 + T_2^2 l - T_2^2 l}{2(l-l_1)}$$

$$= \frac{T_1^2\,(l+l_1) + T_1^2\,(l-l_1) + T_2^2\,(l-l_1) - T_2^2\,(l+l_1)}{2(l-l_1)}$$

$$= \frac{(T_1^2 - T_2^2)(l+l_1) + (T_1^2+T_2^2)(l-l_1)}{2(l-l_1)}$$

$$= \left[\frac{T_1^2+T_2^2}{2}\right] + \left[\frac{T_1^2-T_2^2}{2}\right]\left[\frac{l+l_1}{l-l_1}\right] \quad \text{----------(5)}$$

The quantity on RHS of eq^n(5) is substituted as T_2, where T is called as computed time to the pendulum.

The required eq^n is

$$\frac{4\pi^2(l+l')}{g} = T^2 \text{ --------- (6)}$$

From eq^n (5) $T^2 = \left[\frac{T_1^2+T_2^2}{2}\right] + \left[\frac{T_1^2-T_2^2}{2}\right]\left[\frac{l+l_1}{l-l_1}\right]$ ----- (7)

Quantities T_1 and T_2 are measured with sufficient accuracy. *$(l+l_1)$* also measured accurately, it is distance between K_1 and K_2. But *$(l-l_1)$* the difference between the distances of the two axes from center of gravity cannot be very accurately determined. Even then this error is not too large because $\left[\frac{T_1^2-T_2^2}{2}\right]$ is very small, T_1 being nearly equal T_2. Thus Bessel's formula for computed time helps to calculate value of *g* to sufficient accuracy by using keter's pendulum.

We have $\frac{4\pi^2(l+l')}{g} = \frac{T_1^2+T_2^2}{2}$

$$g = \frac{8\pi^2(l+l_1)}{T_1^2+T_2^2}$$

The value of *g* can be easily calculated.

Numerical: 1) Find the period of compound pendulum suspended by rigid support using a string of length 20cm attached by bob of mass 50gm, rotating along axis having moment of inertia 10kgm^2.

Solution: Given

$l = 20cm = 0.2m$

m=50gm=50×10^{-3}*, I = 10Kg-m*^{2}*, T=?*

Time period of compound pendulum,

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

$$T = 2\times 3.14\sqrt{\frac{10}{50\times 10^{-3}\times 9.8\times 0.2}}$$

$$T = 6.28\sqrt{\frac{10^3}{9.8}} = 6.28\sqrt{102.04}$$

$$T = 6.28\times 10.1015 = 63.44\ s$$

2) A circular disc of diameter 30cm oscillates about transverse axis passing through a point at a distance 5cm from center, calculate period of oscillation.

Solution: Given

2R=30cm

R = 15cm

l=5cm

T=?

Period of compound pendulum

$$T = 2\pi\sqrt{\frac{K^2 + l^2}{gl}}$$

But For circular disc, M. I. of disc passing through $I = \frac{MR^2}{2}$

But $I = MK^2 = \frac{MR^2}{2}$

$$K^2 = \frac{R^2}{2} = \frac{15^2}{2} = \frac{225}{2} = 112.5$$

$$T = 2\pi\sqrt{\frac{K^2 + l^2}{gl}} = 2 \times 3.14\sqrt{\frac{112.5 + 25}{980 \times 5}} = 6.28\sqrt{\frac{137.5}{4900}}$$

$$T = 6.28\sqrt{0.0281} = 6.28 \times 0.1676 = 1.0525\text{s}$$

Gravitation

1. Newton's law of gravitation: In 1687 Newton were explained the force between two particles called as Newton's law of gravitation.

Statement: Every particle of matter in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to square of distance between them.

Let m_1 and m_2 be masses of two particles and r are distance between them.

By Newton's law of gravitation,

Force of attraction between them is

$$F \, \alpha \frac{m_1 . m_2}{r^2}$$

$$F = G\frac{m_1 . m_2}{r^2}$$

Where G is constant of proportionality called as universal gravitational constant.

If $m_1 = m_2 = 1$ gm and $r = 1$ cm then $F = G$.

Thus gravitational constant is equal to the force of attraction between two unit masses of matter, having unit distance apart.

$$G = \frac{F . r^2}{m_1 m_2}$$

The value of G is 6.67×10^{-8} CGS units and its dimension is $[M^{-1}L^3T^{-2}]$.

2. Intensity of gravitational field:

The area round a body, within which it experiences gravitational force of attraction, is called as gravitational field.

The strength of gravitational field at any point is expressed in vector quantity called as gravitational field intensity.

The gravitational field intensity at any point is defined as the force experienced by a unit mass, placed at that point in the field.

∴ Gravitational field intensity is,

$$I = \frac{F}{m} = \frac{GMm}{r^2} \times \frac{1}{m}$$

$$I = \frac{GM}{r^2}$$

Also $F=mg$ where g is acceleration due to gravity,

$$g = \frac{F}{m} = \frac{GM}{r^2} = I$$

It may also be defined as the rate of change of gravitational potential with distance or the potential gradient.

$$I = -\frac{dV}{dx}$$

Where dV is small change of potential for a small distance dx.

3. Gravitational Potential: The strength of gravitational field is expressed in scalar quantity called as gravitational potential.

The gravitational potential is defined as the work done in moving a unit mass of a body from infinity to any point in the gravitational field of a body.

The gravitational potential is,

$$V = \frac{workdone}{mass}$$

$$= -\frac{GM}{r}$$

-ve sign indicates that as distance increases the potential decreases.

Gravitational potential of mass:

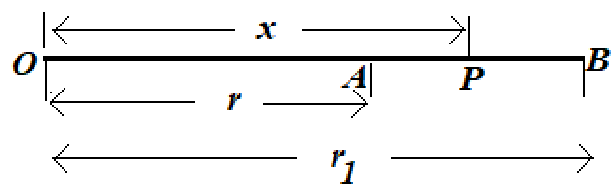
The gravitational potential of a body of mass *m* is expressed as

$$V = -\frac{GMm}{r}$$

This represents the potential energy of a body of mass *m*.

4. Gravitational potential at a point distance *r* from a body of mass *m*:

Let *m* be mass of a body situated at point O and unit mass be situated at point P at a distance *x* from point O.



The force of attraction on the unit mass due to mass *m* is,

$$F = G\frac{m \times 1}{x^2} = \frac{Gm}{x^2}$$

The force being directed towards O. Therefore, work done when the unit mass moves through a small distance *dx* towards O is,

$$work\ done = F.dx = \frac{Gm}{x^2}.dx$$

Therefore, total work done when it moves from B to A is,

$$W = \int_B^A \frac{Gm}{x^2} . dx = \int_{r_1}^{r} \frac{Gm}{x^2} . dx$$

$$= Gm \int_{r_1}^{r} \frac{1}{x^2} . dx$$

$$= Gm \left[-\frac{1}{x} \right]_{r_1}^{r} = -Gm \left[\frac{1}{r} - \frac{1}{r_1} \right]$$

Where r and r1 are distances of A and B from point O. This represents the potential difference between the points A and B.

If B be at infinity i.e. if $r_1 = \infty$, we have,

Potential difference between the points A and B is

$$= -Gm \left[\frac{1}{r} - \frac{1}{\infty} \right] = -\frac{Gm}{r}$$

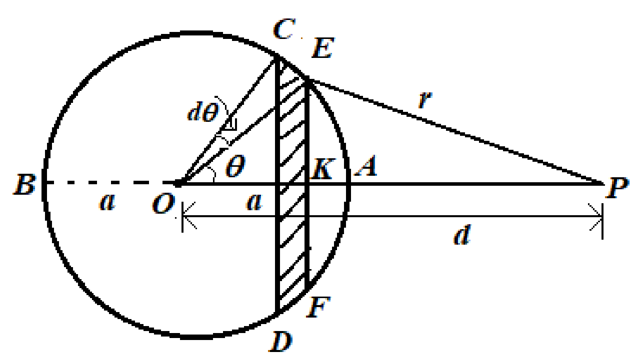
$\therefore$ The gravitational potential at A due to mass m is,

$$V = -\frac{Gm}{r}$$

5. Gravitational potential due to a spherical Shell:

i) At a point outside the spherical Shell:

Consider a spherical shell of radius *a* and centre O is as shown in figure. Let M be mass of spherical shell.



Let P be a point at a distance *d* outside the spherical shell from point O.

Let ρ be surface density i.e. mass per unit area of the surface.

Join OP and cutout a slice CEFD in form of ring by two planes close to each other and perpendicular to radius OA, meeting the shell in C and D and in E and F respectively. Let ∠EOP = θ and ∠COE = $d\theta$.

The radius of the ring is $EK = OE\ sin\theta = a\ sin\theta$

Circumference of ring = $2\pi.EK = 2\pi.asin\theta$

Width of ring =$CE = ad\theta$

$\therefore$ Area of ring or slice = its circumference × its width

$$=2\pi a\ sin\theta \times ad\theta = 2\pi a^2\ sin\theta\, d\theta$$

Its mass = $2\pi a^2\ sin\theta\, d\theta \times \rho$

If EP=r , the potential at point P due to small slice is

$$dV = \frac{-mass\ of\ slice\ \times G}{r} = \frac{-2\pi a^2\ sin\theta d\theta \times \rho G}{r}$$

----

(1)

In ΔOEP, $EP^2 = OE^2 + OP^2 - 2OE.OP\ cos\theta$

$$r^2 = a^2 + d^2 - 2ad\ cos\theta$$

Differentiating with respect to θ

$$\therefore 2rdr = 0+0+2adsin\theta\, d\theta = 2adsin\theta\, d\theta$$

$$r = \frac{adsin\theta d\theta}{dr}$$

Substituting the value of r in eq[n](1)

$$dV = -2\pi a^2 sin\theta d\theta \rho G \times \frac{dr}{adsin\theta d\theta} = \frac{-2\pi a\rho G dr}{d}$$

-------------------------(2)

To find the potential due to whole shell at point P, integrating eq[n](2) between the limits r = AP = d-a and r = BP = d+a

$\therefore$

$$V = \int_{d-a}^{d+a} \frac{-2\pi a\rho G dr}{d} = \frac{-2\pi a\rho G}{d}\int_{d-a}^{d+a} dr$$

$\therefore$

$$V = \frac{-2\pi a\rho G}{d}[r]_{d-a}^{d+a} = \frac{-2\pi a\rho G}{d}[d + a - d + a] = \frac{-2\pi\rho Ga}{d}.2a$$

$$V = -\frac{4\pi a^2 \rho G}{d}$$

Let $4\pi a^2$ be surface area of whole shell and ρ be mass per unit area i.e.

$$\rho = \frac{M}{4\pi a^2} \quad \therefore M = \rho.4\pi a^2$$

$$V = \frac{-GM}{d}$$

This is expression of gravitational potential of at a point outside a spherical shell.

ii) At a point on the surface of the shell:

Suppose point P is on the surface of the shell i.e. at point A, we obtain the potential by integrating eq^n (2) between the limit $r = 0$ and $r = 2a$,

$$V = \int_0^{2a} \frac{-2\pi a\rho G dr}{d} = \frac{-2\pi a\rho G}{d}\int_0^{2a} dr$$

$\therefore$

$$V = \frac{-2\pi a\rho G}{d}[r]_0^{2a} = \frac{-2\pi a\rho G}{d}[2a - 0] = \frac{-2\pi\rho Ga}{d}.2a$$

$$V = -\frac{4\pi a^2 \rho G}{d}$$

Let $4\pi a^2$ be surface area of whole shell and ρ be mass per unit area i.e.

$$\rho = \frac{M}{4\pi a^2} \quad \therefore M = \rho.4\pi a^2$$

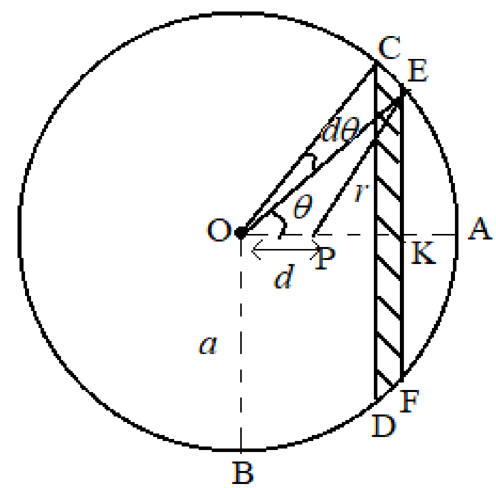
$$V = \frac{-GM}{d}$$

Here $d=a$, the gravitational potential on the surface of spherical shell is,

$$V = \frac{-GM}{a}$$

iii) At a point inside the spherical Shell:

Consider a spherical shell of radius a and centre O is as shown in figure. Let M be mass of spherical shell.



Let P be a point at a distance d inside the spherical shell from point O.

Let ρ be surface density i.e. mass per unit area of the surface.

Join OP and cutout a slice CEFD in form of ring by two planes close to each other and perpendicular to radius OA, meeting the shell in C and D and in E and F respectively. Let $\angle$EOP = θ and $\angle$COE = $d\theta$.

The radius of the ring is $EK = OE \sin\theta = a \sin\theta$

Circumference of ring = $2\pi.EK = 2\pi.a\sin\theta$

Width of ring $= CE = ad\theta$

$\therefore$ Area of ring or slice = its circumference × its width

$$= 2\pi a \sin\theta \times ad\theta = 2\pi a^2 \sin\theta\, d\theta$$

Its mass = $2\pi a^2 \sin\theta\, d\theta \times \rho$

If EP = r , the potential at point P due to small slice is

$$dV = \frac{-mass\ of\ slice\ \times G}{r} = \frac{-2\pi a^2 \sin\theta\, d\theta \times \rho G}{r}$$

---------------------- (1)

In ΔOEP, $$EP^2 = OE^2 + OP^2 - 2OE.OP\cos\theta$$

$$r^2 = a^2 + d^2 - 2ad\cos\theta$$

Differentiating with respect to θ

$$\therefore 2rdr = 0+0+2adsin\theta\, d\theta = 2adsin\theta\, d\theta$$

$$r = \frac{adsin\theta\, d\theta}{dr}$$

Substituting the value of r in eq^n(1)

$$dV = -2\pi a^2 sin\theta d\theta\rho G \times \frac{dr}{adsin\theta d\theta} = \frac{-2\pi a\rho Gdr}{d}$$

-------------------------(2)

To find the potential due to whole shell at point P, integrating eq^n(2) between the limits $r = a\text{-}d$ and $r = a+d$

$$V = \int_{a-d}^{a+d} \frac{-2\pi a\rho Gdr}{d} = \frac{-2\pi a\rho G}{d}\int_{a-d}^{a+d} dr$$

$\therefore$

$$V = \frac{-2\pi a\rho G}{d}[r]_{a-d}^{a+d} = \frac{-2\pi a\rho G}{d}[a+d-a+d] = \frac{-2\pi\rho Ga}{d}.2d$$

$$V = -4\pi a\rho G$$

Multiplying and dividing by a, we get,

$$V = -\frac{4\pi a^2\rho G}{a}$$

But $4\pi a^2\rho = M$ be mass of shell.

$$V = \frac{-GM}{a}$$

This is expression of gravitational potential at a point inside the spherical shell i.e. gravitational potential is same as at a point on the shell.

6. Gravitational Field due to a spherical Shell:

i) At a point outside the shell:-

We know that the gravitational field at a point is given by the potential gradient at that point. $I = -\frac{dV}{dx}$

The gravitational potential at a point at a distance x from center of the shell is,

$$V = \frac{-GM}{x}$$

Therefore, intensity of gravitational field at that point is,

$$I = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{-GM}{x}\right)$$

$$I = -\frac{GM}{x^2}$$

ii) At a point inside the shell:-

We know that the gravitational field at a point is given by the potential gradient at that point. $I = -\frac{dV}{dx}$

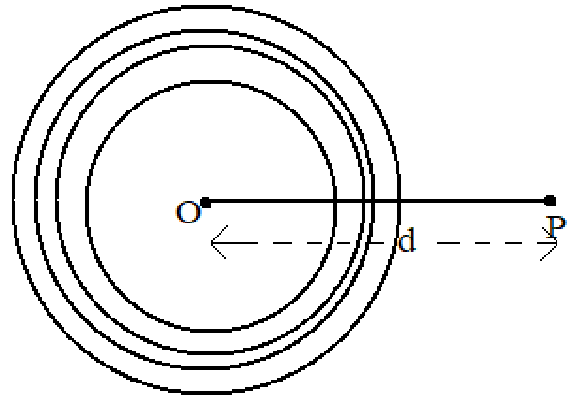
The gravitational potential at all point inside a spherical shell is same. Since V is constant for all the points inside the shell i.e. $\frac{dV}{dx} = 0$

Therefore, there is no gravitational field inside a spherical shell.

7. Gravitational potential due to a solid sphere:

i) At a point outside the solid sphere:

Let P be a point outside the solid sphere at a distance *d* from center of sphere as shown in figure.



Let M be mass and *a* be radius of solid sphere.

Imagine the sphere consist of large number of thin shells, concentric with the sphere.

Let m_1, m_2, m_3, ------- etc be masses of sphere

The gravitational potential at P due to each spherical shell is,

$$= -\frac{mass \times G}{d}$$

The potential at P due to different shells will be, $-m_1G/d$, $-m_2G/d$, $-m_3G/d$ and soon.

The potential at P due to all such shells i.e. due to whole solid sphere is,

$$V = -\left[\frac{m_1 G}{d} + \frac{m_2 G}{d} + \frac{m_3 G}{d} + \dots\dots\dots\dots\dots..\right]$$

$$= -(m_1 + m_2 + m_3 + \dots\dots)\frac{G}{d}$$

$$V = -\frac{MG}{d}$$

where $m_1+m_2+m_3+$-------- = M be the mass of the solid sphere.

Hence, gravitational potential at P outside the solid sphere is,

$$V = -\frac{MG}{d}$$

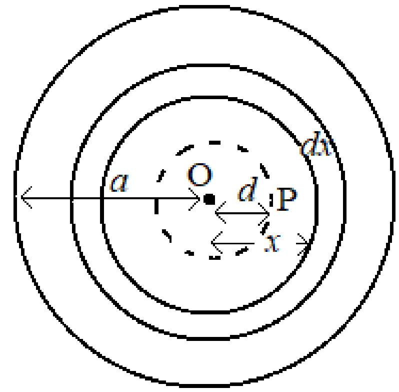
ii) At a point on the surface of the sphere:

If $d=a$, we have, potential at the surface of the sphere is

$$V = -\frac{MG}{a}$$

iii) At a point inside the sphere:

Consider a solid sphere of radius *a* and center O as shown in figure.



Let M be mass of sphere.

Let P be a point inside a solid sphere at a distance *d* from centre O.

Let σ be volume density i.e. mass per unit volume

Therefore,

$$\sigma = \frac{mass}{Volume\ of\ sphere} = \frac{M}{V}$$

$$M = \sigma \times \frac{4}{3}\pi a^3 \qquad \text{---------------------------- (1)}$$

The solid sphere may be imagined to be made up of an inner solid sphere of radius *d*, surrounded by a number of hollow spheres concentric with it. The potential at P due to solid sphere is equal to the sum of the potentials at P due to inner solid sphere and all such spherical shells outside it.

∴ the potential at P due to the sphere of radius *d* is,

$$V_1 = \frac{-mass\ of\ sphere \times G}{d}$$

But , $Density = \frac{mass}{volume}$

$$mass = volume\ \times density = \sigma \times \frac{4}{3}\pi d^3$$

$$\therefore V_1 = \frac{-\frac{4\pi d^3 \sigma}{3}}{d} G = -\frac{4}{3}\pi d^2 \sigma G$$

-------------- (2)

To determine potential at P due to outer shells, imagine a single shell of radius x and thickness dx.

∴ its volume = area × thickness = $4\pi x^2.dx$

And its mass = $4\pi x^2.dx\sigma$

The potential at any point within the shell is the same as at any other point on its surface.

$$Potential\ at\ P\ due\ to\ this\ shell = -\frac{4\pi x^2 dx\sigma.G}{x} = -4\pi\sigma Gxdx$$

Integrating both side with limits $x=d$ and $x=a$, therefore potential at P due to all shells,

$$V_2 = \int_d^a -4\pi\sigma Gxdx = -4\pi\sigma G\left[\frac{x^2}{2}\right]_d^a$$

$$= -4\pi\sigma G\left[\frac{a^2-d^2}{2}\right]$$

-------------(3)

The total potential at P due to whole solid sphere is equal to the potential at P due to inner sphere of radius d plus the potential at P due to all outer shells.

∴ total potential at P is,

$$V = V_2 + V_3$$

Substituting value of V_2 and V_3 from eq^n (2) and (3),

$$V = -\frac{4}{3}\pi d^2\sigma G - 4\pi\sigma G\left[\frac{a^2-d^2}{2}\right]$$

$$= -\frac{4}{3}\pi d^2\sigma G - \frac{4}{3}\pi\sigma G\left[\frac{3a^2-3d^2}{2}\right]$$

$$= -\frac{4\pi\sigma G}{3}\left[d^2 + \frac{3a^2 - 3d^2}{2}\right]$$

$$= -\frac{4\pi\sigma G}{3}\left[\frac{2d^2 + 3a^2 - 3d^2}{2}\right]$$

$$= -\frac{4\pi\sigma G}{3}\left[\frac{3a^2 - d^2}{2}\right]$$

Multiplying and dividing by a^3

$$V = -\frac{4\pi\sigma G a^3}{3}\left[\frac{3a^2 - d^2}{2a^3}\right]$$

From eq^n (1) $M = \sigma \times \frac{4}{3}\pi a^3$ mass of sphere.

∴ Potential at P inside the sphere is

$$V = -GM\left[\frac{3a^2 - d^2}{2a^3}\right]$$

8. Gravitational field due to a solid sphere:

i) At a point outside the sphere:-

We know that the gravitational field at a point is given by the potential gradient at that point. $I = -\frac{dV}{dx}$

The gravitational potential at a point at a distance x from center of the sphere is,

$$V = \frac{-GM}{x}$$

Therefore, intensity of gravitational field at that point is,

$$I = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{-GM}{x}\right)$$

$$I = -\frac{GM}{x^2}$$

ii) At a point inside the sphere:-

We know that the gravitational field at a point is given by the potential gradient at that point. $I = -\frac{dV}{dx}$

The gravitational potential at a point at a distance x inside the solid sphere is,

$$V = -GM\left[\frac{3a^2 - x^2}{2a^3}\right]$$

Therefore, intensity of gravitational field at that point is,

$$I = -\frac{dV}{dx} = -\frac{d}{dx}\left(-GM\left[\frac{3a^2 - x^2}{2a^3}\right]\right)$$

$$I = GM\left(\frac{0 - 2x}{2a^3}\right)$$

$$I = -\frac{MGx}{a^3}$$

i.e. the intensity of the field is directly proportional to the distance from centre of the sphere.

Numerical:

1) A sphere of a mass 40kg is attached by another sphere of mass 80kg with a force 0.01×10^{-5} N. Find distance between centers of two spheres. (G = 6.67×10^{-11} SI units).

Solution: Given, m_1= 40 kg, m_2 = 80kg

F= 0.01×10^{-5} N= 10^{-7}

G= 6.67×10^{-11}

r =?

Gravitational force between two spheres is,

$$F = \frac{Gm_1m_2}{r^2}$$

$$r^2 = \frac{Gm_1m_2}{F} = \frac{6.67 \times 10^{-11} \times 40 \times 80}{10^{-7}} = 6.67 \times 3200 \times 10^{-4}$$

$$r^2 = 6.67 \times 32 \times 10^{-2} = 213.44 \times 10^{-2} = 2.1344$$

$$r = \sqrt{2.1344} = 1.4609m$$

2) The radius of the moon is 1.7×10^6m, its mass is 7.35×10^{22} kg. Find the acceleration due to gravity on Moon surface. ($G = 6.67\times10^{-11}$ SI units).

Solution: Given

$R= 1.7\times10^6$m

$M= 7.35\times10^{22}$ kg

$G = 6.67\times10^{-11}Nm^2/kg^2$

Acceleration due to gravity on moon surface,

$$g = \frac{GM}{R^2}$$

$$g = \frac{6.67\times10^{-11}\times7.53\times10^{22}}{(1.7\times10^6)^2} = \frac{6.67\times7.53\times10^{-1}}{2.89}$$

$$g = 16.96\times10^{-1} = 1.696 m/s^2$$

3) Find the acceleration due to gravity at surface of moon. Given that the mass of moon is 1/80th that of the earth and radius of moon is ¼ th that of the earth and acceleration due to gravity at surface of earth is $9.8 m/s^2$.

Solution: Given

$g_m = ?$, $M_m = 1/80\ M_e$, $R_m = ¼\ R_e$

Acceleration due to gravity

$$g = \frac{GM}{R^2}$$

Acceleration due to gravity on the surface of earth

$$g_e = \frac{GM_e}{{R_e}^2}$$

Acceleration due to gravity on the surface of moon is,

$$g_m = \frac{GM_m}{{R_m}^2}$$

$\therefore$

$$\frac{g_e}{g_m} = \frac{GM_e}{{R_e}^2} \times \frac{R_m^2}{GM_m} = \frac{M_e}{\frac{1}{80}Me} \times \left(\frac{\frac{1}{4}R_e}{R_e}\right)^2$$

$$\frac{g_e}{g_m} = 80 \times \frac{1}{16} = 5$$

$$g_m = \frac{g_e}{5} = \frac{9.8}{5} = 1.96\, m/s^2$$

4) Calculate gravitational field intensity and potential at a distance 4 times radius of the earth from its centre. Assuming that earth is homogeneous solid sphere. [Radius of the earth =6400Km, Mass of the earth = 6×10^{24}kg , G = 6.67×10^{-11} Nm^2/Kg^2.

Solution: Given

R= 6400Km = 6.4×10^6m

M= 6×10^{24} kg

G = $6.67\times10^{-11} Nm^2/kg^2$

r = 4R =$4\times6.4\times10^6$= 25.6×10^6 m

Gravitational field intensity at a distance *r* is given by,

$$I = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(25.6 \times 10^6)^2}$$

$$\mathrm{I} = \frac{40.02 \times 10^{13}}{655.36 \times 10^{12}} = \frac{400.2}{655.36} = 0.6106 \text{ N/kg}$$

Gravitational potential at a distance r is given by

$$V = -\frac{GM}{r} = -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{4 \times 6.4 \times 10^6}$$

$$= \frac{40.02 \times 10^{13}}{25.6 \times 10^6} = 1.563 \times 10^7 J/Kg$$

5) Calculate gravitational potential and gravitational field intensity at a point 10cm from the centre of a uniform solid sphere of mass 10kg and diameter 40cm.

Solution: Given

r = 10cm =10×10^{-2}=0.1 m

m=10 kg

2*a*= 40cm =0.40m

a = 0.2m

G = 6.67×10^{-11}Nm^2/kg^2

Since $r\angle a$ i.e. inside the sphere

Gravitational potential at a distance *r* from center of sphere is,

$$V = -Gm\left[\frac{3a^2 - r^2}{2a^3}\right]$$

$$V = -6.67 \times 10^{-11} \times 10\left[\frac{3 \times 0.2^2 - 0.1^2}{2 \times 0.2^3}\right]$$

$$V = -6.67 \times 10^{-10}\left[\frac{0.12 - 0.01}{0.016}\right]$$

$$V = -6.67 \times 10^{-10}\left[\frac{0.11}{0.016}\right]$$

$$V = -6.67 \times 10^{-10} \times 6.875 = -45.85 \times 10^{-10} = -4.585 \times 10^{-9} J/Kg$$

Gravitational field intensity is,

$$I = \frac{GMr}{a^3} = \frac{6.67 \times 10^{-11} \times 10 \times 0.1}{(0.2)^3}$$

$$I = \frac{6.67 \times 10^{-11}}{0.008} = \frac{6.67 \times 10^{-8}}{8} = 0.8337 \times 10^{-8}$$

$$I = 8.337\times10^{-9} \text{ N/kg}$$

Multiple Choice Questions:

1) Time period of oscillation of compound pendulum is depends upon

a) Moment of inertia b) mass of pendulum

c) Length of pendulum **d) All of these**

2) Compound pendulum is a rigid body capable of freely oscillating about ------

a) **Horizontal axis** b) Vertical axis

c) sometimes horizontal axis d) sometimes vertical axis.

3) If the mass of the object is doubled then what will be the effect of time period of the compound pendulum?

a) Doubled b) Remains same c) Halved **d) Decreases by √2 times**

4) Calculate the time period of an object having moment of inertia = 100 Kg-m^2, mass of 10 Kg and the centre of gravity lies at a point 20 cm below the point of suspension.

a) 14.1 b) 15.2 c) 13.3 d) 12.9

5. By interchanging center of suspension and center of oscillation, the time period of compound pendulum is ------

a) Changes **b) remains same**

c) first increases then decreases d) first decreases then decreases

6. Keters pendulum can be used to determine the value of ------

a) mass of an object b) length of an object

c) acceleration due to gravity d) Gravitational constant

7. If m_1 and m_2 are the masses of the two particles separated by a distance r, and universal gravitational constant G, the force of gravitational attraction is given by,

a) $\boldsymbol{F = G\frac{m_1.m_2}{r^2}}$ b) $F = G\frac{r^2}{m_1 m_2}$

c) $F = \frac{r^2}{G\, m_1 m_2}$ d) $F = \frac{m_1.m_2}{Gr^2}$

8. In Newton's law of gravitation, the force is inversely proportional to,

a) distance between two particles

b) root of distance between two particles

c) square of distance between two particles

d) half of distance between two particles

9. The strength of gravitational field represented by using vector quantity is—

a) **Gravitational Field intensity** b) gravitational potential

c) gravitational mass d) acceleration

10. SI unit of gravitational potential is

a) J b) J.Kg c) Kg-m **d) J/Kg**

11. Gravitational potential at a point outside the spherical shell is---

a) $\boldsymbol{\frac{-GM}{r}}$ b) $\frac{-GM}{r^2}$ c) $\frac{-gM}{r}$ d) $\frac{-GM}{r^3}$

12. Gravitational field intensity inside the spherical shell is,

a) $\frac{-GM}{r^2}$ **b) zero** c) $\frac{-GM}{r^3}$ d) $\frac{-GM}{r}$

13. Gravitational potential inside the spherical shell at every point is,

a) remains same b) increases from centre to outward

c) decreases from centre to outward d) Zero

14. Gravitational potential at a point inside the solid sphere of radius *a* and at a distance *r* from center of sphere is-----

a) $\boldsymbol{V = -GM\left[\frac{3a^2-r^2}{2a^3}\right]}$ b) $V = -GM\left[\frac{a^2-r^2}{2a^3}\right]$

c) $V = -\frac{GM}{a^2}$ d) $V = -\frac{GM}{a}$

15. Gravitational field intensity at a point inside the solid sphere of radius *a* and at a distance *r* from center of sphere is-----

a) $-\frac{GM}{r^2}$ **b)** $-\frac{\boldsymbol{GMr}}{\boldsymbol{a^3}}$ c) $-\frac{GMa^3}{r}$ d) $-GMa^4$

16. A sphere of a mass 10kg is attached by another sphere of mass 20kg keeping a distance between them 0.2m. Then gravitational force between them is (G = 6.67×10^{-11} SI units).

a) 3.335×10^{-7} N b) 3.335×10^{-7} dyne

c) 3.335×10^{7} N d) 3.335×10^{7} dyne

17. The gravitational potential at a point outside the solid sphere of mass 2kg from centre of sphere at a distance of 5cm is ---

a) -3.668 $\times10^{-7}$ J/Kg **b) -2.668 $\times10^{-9}$ J/Kg**

c) -4.668 $\times10^{-9}$ J/Kg d) -2.668 $\times10^{9}$ J/Kg

18. Dimension of Gravitational Constant are-----

a) $[M^{-1}L^3T^2]$ **b) $[M^{-1}L^3T^{-2}]$** c) $[M^1L^3T^2]$ d) No dimension

19. Relation between gravitational field intensity and gravitational potential is

a) $\boldsymbol{I = -\frac{dV}{dx}}$ b) $I = -\frac{dx}{dV}$ *c) I=-dV* *d) I=-dx*

20. The gravitational field intensity of a body of mass 20kg, 20cm from its centre is ---- (G = 6.67×10^{-11} SI units)

a) 3.335×10^{-6} N/kg **b) 3.335×10^{-8} N/kg**

c) 6.335×10^{-8} N/kg d) 3.335×10^{8} N/kg.